MATH II CCR MATH STANDARDS

Mathematical Habits of Mind

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

RELATIONSHIPS BETWEEN QUANTITIES

Cluster	**Extend the properties of exponents to rational exponents.**				
M.2HS.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. (e.g., We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.)				
M.2HS.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.				
Cluster	**Use properties of rational and irrational numbers.**				
M.2HS.3	Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations, e.g., finding the perimeter of a square of area 2.				
Cluster	**Perform arithmetic operations with complex numbers.**				
M.2HS.4	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.				
M.2HS.5	Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. Instructional Note: Limit to multiplications that involve i^2 as the highest power of i				
Cluster	**Perform arithmetic operations on polynomials.**				
M.2HS.6	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials. Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.				

QUADRATIC FUNCTIONS AND MODELING

Cluster	**Interpret functions that arise in applications in terms of a context.**				
M.2HS.7	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.				
M.2HS.8	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.				
M.2HS.9	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.				
Cluster	**Analyze functions using different representations.**				
	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.				
	a. Graph linear and quadratic functions and show intercepts, maxima, and minima.				

M.2HS.10	b. Graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range and usefulness when examining piecewise-defined functions.				
	Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.				
M.2HS.11	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.				
	a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values and symmetry of the graph and interpret these in terms of a context.				
	b. Use the properties of exponents to interpret expressions for exponential functions. (e.g., Identify percent rate of change in functions such as $y = (1.02)^{t,}$ $y = (0.97)^{t}$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.) Instructional Note: This unit and, in particular, this standard extends the work begun in Mathematics I on exponential functions with integer exponents.				
	Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.				
M.2HS.12	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum). Instructional Note: Focus on expanding the types of functions considered to include, linear, exponential and quadratic. Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.				
Cluster	**Build a function that models a relationship between two quantities.**				
M.2HS.13	Write a function that describes a relationship between two quantities.				
	a. Determine an explicit expression, a recursive process or steps for calculation from a context.				
	b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic or exponential relationship.				
Cluster	**Build new functions from existing functions.**				
M.2HS.14	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions and consider including absolute value functions.				
M.2HS.15	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x)= 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x > 0$.				
Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**				
M.2HS.16	Using graphs and tables, observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically; or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth studied in Mathematics I to quadratic growth.				

EXPRESSIONS AND EQUATIONS

Cluster	**Interpret the structure of expressions.**				
M.2HS.17	Interpret expressions that represent a quantity in terms of its context.				
	a. Interpret parts of an expression, such as terms, factors, and coefficients.				
	b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.				

	Instructional Note: Focus on quadratic and exponential expressions. Exponents are extended from the integer exponents found in Mathematics I to rational exponents focusing on those that represent square or cube roots.				
M.2HS.18	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. Instructional Note: Focus on quadratic and exponential expressions.				
Cluster	**Write expressions in equivalent forms to solve problems.**				
M.2HS.19	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.				
	a. Factor a quadratic expression to reveal the zeros of the function it defines.				
	b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.				
	c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.				
	Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.				
Cluster	**Create equations that describe numbers or relationships.**				
M.2HS.20	Create equations and inequalities in one variable and use them to solve problems. Instructional Note: Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Extend work on linear and exponential equations in Mathematics I to quadratic equations.				
M.2HS.21	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Extend work on linear and exponential equations in Mathematics I to quadratic equations.				
M.2HS.22	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Extend to formulas involving squared variables. Extend work on linear and exponential equations in Mathematics I to quadratic equations.				
Cluster	**Solve equations and inequalities in one variable.**				
M.2HS.23	Solve quadratic equations in one variable.				
	a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.				
	b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.				
	Instructional Note: Extend to solving any quadratic equation with real coefficients, including those with complex solutions.				
Cluster	**Use complex numbers in polynomial identities and equations.**				
M.2HS.24	Solve quadratic equations with real coefficients that have complex solutions. Instructional Note: Limit to quadratics with real coefficients.				
M.2HS.25 (+)	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. Instructional Note: Limit to quadratics with real coefficients.				
M.2HS.26 (+)	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Instructional Note: Limit to quadratics with real coefficients.				
Cluster	**Solve systems of equations**.				
M.2HS.27	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (e.g., Find the points of intersection between the line y = –3x and the circle $x^2 + y^2 = 3$.) Instructional Note: Include systems that lead to work with fractions. (e.g., Finding the intersections between $x^2 + y^2 = 1$ and y = (x+1)/2 leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.)				

APPLICATIONS OF PROBABILITY

Cluster	Understand independence and conditional probability and use them to interpret data.				
M.2HS.28	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes or as unions, intersections or complements of other events ("or," "and," "not").				
M.2HS.29	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.				
M.2HS.30	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.				
M.2HS.31	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. (e.g., Collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.) Instructional Note: Build on work with two-way tables from Mathematics I to develop understanding of conditional probability and independence.				
M.2HS.32	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. (e.g., Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.)				
Cluster	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**				
M.2HS.33	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.				
M.2HS.34	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.				
M.2HS.35 (+)	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.				
M.2HS.36 (+)	Use permutations and combinations to compute probabilities of compound events and solve problems.				
Cluster	**Use probability to evaluate outcomes of decisions.**				
M.2HS.37 (+)	Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator).				
M.2HS.38 (+)	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: This unit sets the stage for work in Mathematics III, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e., incomplete information) requires an understanding of probability concepts.				

SIMILARITY, RIGHT TRIANGLE TRIGONOMETRY, AND PROOF

Cluster	Understand similarity in terms of similarity transformations				
M.2HS.39	Verify experimentally the properties of dilations given by a center and a scale factor.				
	a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.				
	b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.				
M.2HS.40	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.				
M.2HS.41	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.				
Cluster	**Prove geometric theorems.**				

M.2HS.42	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Implementation may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for M.2HS.C.3. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.				
M.2HS.43	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of this standard may be extended to include concurrence of perpendicular bisectors and angle bisectors in preparation for the unit on Circles With and Without Coordinates.				
M.2HS.44	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.				
Cluster	**Prove theorems involving similarity.**				
M.2HS.45	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally and conversely; the Pythagorean Theorem proved using triangle similarity.				
M.2HS.46	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.				
Cluster	**Use coordinates to prove simple geometric theorems algebraically.**				
M.2HS.47	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.				
Cluster	**Define trigonometric ratios and solve problems involving right triangles.**				
M.2HS.48	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.				
M.2HS.49	Explain and use the relationship between the sine and cosine of complementary angles.				
M.2HS.50	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.				
Cluster	**Prove and apply trigonometric identities.**				
M.2HS.51	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle. Instructional Note: Limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.				

CIRCLES WITH AND WITHOUT COORDINATES

Cluster	**Understand and apply theorems about circles.**				
M.2HS.52	Prove that all circles are similar.				
M.2HS.53	Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.				
M.2HS.54	Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.				

M.2HS.55 (+)	Construct a tangent line from a point outside a given circle to the circle.				
Cluster	**Find arc lengths and areas of sectors of circles.**				
M.2HS.56	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Instructional Note: Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.				
Cluster	**Translate between the geometric description and the equation for a conic section.**				
M.2HS.57	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. Instructional Note: Connect the equations of circles and parabolas to prior work with quadratic equations.				
M.2HS.58	Derive the equation of a parabola given the focus and directrix. Instructional Note: The directrix should be parallel to a coordinate axis.				
Cluster	**Use coordinates to prove simple geometric theorems algebraically.**				
M.2HS.59	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).) Instructional Note: Include simple proofs involving circles.				
Cluster	**Explain volume formulas and use them to solve problems.**				
M.2HS.60	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle and informal limit arguments. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first.				
M.2HS.61	Use volume formulas for cylinders, pyramids, cones and spheres to solve problems. Volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.				